Rate Splitting Approach Under PSK signalling Using Constructive Interference Precoding Technique

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Introduction

- Rate-Splitting (RS) technique has received significant attention very recently, as a viable multiple access technique for wireless communication networks.
- In RS scheme, a user message is split into a common part and a private part, where the common part can be decoded by all the users with zero error probability, while the private part can be decoded only by the corresponding user.
- The private messages are transmitted by MU-precoders such as ZF, while the common message is precoded in a multicast fashion such that it is delivered to all users.
- At the reception, each user decodes firstly the common message, and then decodes its own private message after removing the common message via Successive Interference Cancellation (SIC) technique.

- Recently, constructive interference (CI) precoding technique has been proposed to enhance the performance of downlink MU-MIMO systems.

- The interference signal is considered to be constructive to the transmitted signal if it moves the received symbols away from the decision thresholds of the constellation towards the direction of the desired symbol.

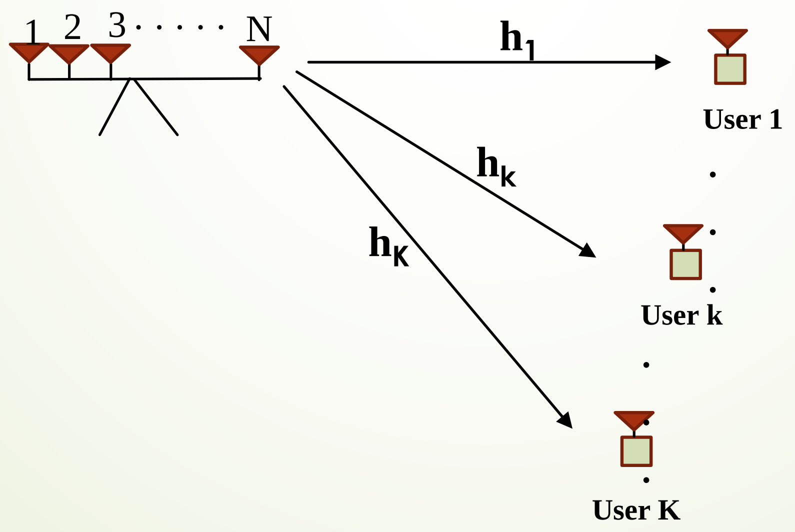
- Consequently, the transmit precoding can be designed such that the resulting interference is constructive to the desired symbol.

Contributions:

- We employ the CI precoding technique to further enhance the sum-rate achieved by RS scheme in MU-MIMO systems under a PSK input alphabet.
- New analytical expressions for the average sum-rate are derived for two precoding techniques of the private messages, namely, 1) CI precoding, 2) ZF precoding.
- Additionally, the conventional transmission without rate splitting is also studied in this work.
- Furthermore, power allocation between the common and private messages is investigated.
- Monte-Carlo simulations are provided throughout our investigations, and the impact of different system parameters on the system performance are investigated.

System Model

- Consider a downlink MU-MIMO system, consisting of a BS equipped with N antennas communicating with K single antenna users.
- All the channels are modelled as independent identically distributed (i.i.d) Rayleigh fading channels.



- In RS, the signal for a user is split into a common part and a private part. The common and the private symbols are denoted by x_c and x_k .
- Therefore, the transmitted signal can be mathematically expressed by

$$\mathrm{s} = \mathrm{W}\mathrm{x} = \sqrt{P_c}\mathrm{w}_c x_c + \sum_{k=1}^{K} \sqrt{P_p}\mathrm{w}_k x_k$$

where $\mathbf{W} = [\mathbf{w}_c, \mathbf{w}_1, ...\mathbf{w}_K]$,$\mathbf{w}_c$ is the common precoder of the common message and $\mathbf{w}_k$ is the kth private precoder, P_c is the power allocated to the common message and P_p is the power allocated to the private message, where $P_c = (1-t)\,P$ and $P_p = \frac{tP}{K}, \; 0 < t \leq 1$ and P is the total power.

- The received signal at the kth user can be written as

$$y_k = \mathbf{h}_k \mathbf{W} \mathbf{x} + n_k = \sqrt{P_c} \mathbf{h}_k \mathbf{w}_c x_c + \sum_{k=1}^{K} \sqrt{P_p} \mathbf{h}_k \mathbf{w}_k x_k + n_k$$

- At the user side, the common symbol is decoded firstly, and then each user decodes its own message after removing the common message using SIC technique.
- The received private signal at the kth user in this system can be written as

$$y_k^p = \sqrt{P_p} \mathbf{h}_k \mathbf{W}^p \mathbf{x}^p + n_k$$

where $\mathrm{x}^p = [x_1, ..., x_K]^T$ and $\mathrm{W}^p = [\mathrm{w}_1, ...\mathrm{w}_K]$.

- The sum rate in this scenario can be expressed by

$$R = R^c + \sum_{k=1}^{K} R_k^p$$

where R^c is the rate for the common part, $R^c = \min\left(R_1^c, R_2^c, .., R_k^c, .., R_K^c\right)$, R_k^c is the rate for the common message at user k, and R_k^p is the rate for the private part at user k.

- The rate for the common and private parts at user k under PSK signalling can be written respectively as

$$R_k^c = \log_2 M - \frac{1}{M^N} \sum_{m=1}^{M^N} \underbrace{\mathcal{E}_{\mathbf{h},n_k} \left\{ \log_2 \sum_{i=1}^{M^N} e^{\frac{-|\mathbf{h}_k \mathbf{W} \mathbf{x}_{m,i} + n_k|^2}{\sigma_k^2}} \right\}}_{\varphi}$$

$$+ \frac{1}{M^{N-1}} \sum_{m=1}^{M^{N-1}} \underbrace{\mathcal{E}_{\mathbf{h},n_k} \left\{ \log_2 \sum_{i=1}^{M^{N-1}} e^{\frac{-\left|\mathbf{h}_k \mathbf{W}_{CI}^p \mathbf{x}_{m,i}^p + n_k\right|^2}{\sigma_k^2}} \right\}}_{\psi},$$

and

$$R_k^p = \log_2 M - \log_2 e$$

$$- \frac{1}{M^N} \sum_{m=1}^{M^N} \underbrace{\mathcal{E}_{\mathbf{h},n_k} \left\{ \log_2 \sum_{i=1}^{M^N} e^{\frac{-\left|\sqrt{P_p}\mathbf{h}_k \mathbf{W}_{CI}^p \mathbf{x}_{m,i}^p + n_k\right|^2}{\sigma_k^2}} \right\}}_{\psi}$$

where $\mathrm{x}_{m,i} = \mathrm{x}_m - \mathrm{x}_i$, x_m and x_i contain symbols taken from the M signal constellation.

Rate Splitting with MRT/CI precoding

- In this scenario MRT technique is used for common message and CI precoding for the private messages.
- The precoders for the common and the private messages can be written, respectively, as

$$\mathbf{w}_c = \sum_{i=1}^{K} \beta_c \mathbf{h}_i^H$$

$$\mathbf{W}_{CI}^{p} = \frac{1}{K} \beta_p \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathrm{diag}\left\{\mathbf{V}^{-1}\mathbf{u}\right\},$$

where β_c and β_p are the scaling factors to meet the transmit power constraint at the BS, and

$$\mathbf{V} = \mathrm{diag}\left(\mathbf{s}^H\right)\left(\mathbf{H}\mathbf{H}^H\right)^{-1} \mathrm{diag}\left(\mathbf{s}\right).$$

- Following the principles of CI, and by invoking Jensen inequality, we can obtain very accurate approximation of the average rates for the common and private parts, respectively, as

$$R_k^c = \log_2 M - \frac{1}{M^N}\sum_{m=1}^{M^N}\log_2\sum_{i=1}^{M^N}\left(1+\frac{P|\xi|^2\varpi_k\Psi_{m,i}}{2\sigma_k^2}\right)^{-N}$$

$$+\frac{1}{M^{N-1}}\sum_{m=1}^{M^{N-1}}\log_2\sum_{i=1}^{M^{N-1}}\left(\left(\frac{2^{\left(\frac{1}{2}(N-K-1)\right)}K^{(N-K+1)}\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|^{-2+K-N}}{(N-K)!}\right)\left(\left(\frac{c^2}{\sigma_k^2}\right)^{\frac{1}{2}(K-N-1)}\right)\right.$$

$$\times\left(\left(c^2\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|\right)\Gamma\left(\frac{1}{2}(N-K+1)\right){}_1\mathrm{F}_1\left(\frac{1}{2}(N-K+1),\frac{1}{2},\frac{K^2\sigma_k^2}{2c^2\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|^2}\right)\right.$$

$$\left.\left.-\sqrt{2}K\,c\,\sigma_k^2\Gamma\left(\frac{1}{2}(N-K+2)\right){}_1\mathrm{F}_1\left(\frac{1}{2}(N-K+2),\frac{3}{2},\frac{K^2\sigma_k^2}{2c^2\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|^2}\right)\right)\right).$$

$$R_k^p = \log_2 M - \frac{1}{M^N}\sum_{m=1}^{M^N}\log_2\sum_{i=1}^{M^N}\left(\left(\frac{2^{\left(\frac{1}{2}(N-K-1)\right)}K^{(N-K+1)}\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|^{-2+K-N}}{(N-K)!}\right)\left(\left(\frac{c^2}{\sigma_k^2}\right)^{\frac{1}{2}(K-N-1)}\right)\right.$$

$$\left(\left(c^2\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|\right)\Gamma\left(\frac{1}{2}(N-K+1)\right){}_1\mathrm{F}_1\left(\frac{1}{2}(N-K+1),\frac{1}{2},\frac{K^2\sigma_k^2}{2c^2\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|^2}\right)\right.$$

$$\left.\left.-\sqrt{2}K\,c\,\sigma_k^2\Gamma\left(\frac{1}{2}(N-K+2)\right){}_1\mathrm{F}_1\left(\frac{1}{2}(N-K+2),\frac{3}{2},\frac{K^2\sigma_k^2}{2c^2\left|\left[\mathrm{x}_{m,i}^p\right]_k\right|^2}\right)\right)\right).$$

where $\xi = \frac{\beta_p}{K}\mathrm{a}_k\left(\mathrm{diag}\left(\mathrm{x}^{pH}\right)\right)^{-1}(\Sigma)\left(\mathrm{diag}\left(\mathrm{x}^p\right)\right)^{-1}\mathrm{u}$ and $\Psi_{m,i} = \left|\frac{\sqrt{(1-t)N\beta_c[\mathbf{x}_{m,i}]_1}}{|\xi|} + \sqrt{t}N\beta_p\left[\mathrm{x}_{m,i}^p\right]_k\right|^2$

Rate Splitting with MRT/ZF precoding

- In this scenario MRT technique is used for common message and ZF precoding for the private messages.
- The precoders for the common and the private messages can be written, respectively, as

$$\mathbf{w}_c = \sum_{i=1}^{K} \beta_c \mathbf{h}_i^H$$

$$\mathbf{W}_{ZF}^{p} = \beta_p \, \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H \right)^{-1}$$

where β_c and β_p are the scaling factors to meet the transmit power constraint at the BS.

- By invoking Jensen inequality, very accurate approximation of the average rates for the common and private parts can be written, respectively as

$$R_k^c = \log_2 M - \frac{1}{M^N} \sum_{m=1}^{M^N} \log_2 \sum_{i=1}^{M^N} \sum_{r=1}^{n} \frac{(y_r)^{v-1} \mathrm{H}_r}{\Gamma(v)} e^{-\frac{P\left|\sqrt{(1-t)}\beta_c \theta y_r [\mathbf{x}_{m,i}]_1 + \sqrt{t}\beta_p [\mathbf{x}_{m,i}^p]_k\right|^2}{2\sigma_k^2}}$$

$$+ \frac{1}{M^{N-1}} \sum_{m=1}^{M^{N-1}} \log_2 \sum_{t=1}^{M^{N-1}} e^{-\frac{\left|\sqrt{tP}\beta_p [\mathbf{x}_{m,i}^p]_k\right|^2}{2\sigma_k^2}}$$

and

$$R_k^p = \log_2 M - \frac{1}{M^N} \sum_{m=1}^{M^N} \log_2 \sum_{t=1}^{M^N} e^{-\frac{\left|\sqrt{tP}\beta_p [\mathbf{x}_{m,i}^p]_k\right|^2}{2\sigma_k^2}}$$

where y_r and H_r are the rth zero and the weighting factor of the Laguerre polynomials, respectively.

Conventional Transmission without RS (NORS)

1. CI: very accurate approximation of the average rate at the kth user in conventional transmission without RS, using CI precoding can be written as

$$R_k^{NoRS} = \log_2 M - \frac{1}{M^N}\sum_{m=1}^{M^N} \log_2 \sum_{i=1}^{M^N} \left(\left(\frac{2^{\left(\frac{1}{2}(N-K-1)\right)} K^{(N-K+1)} \left| \left[\mathbf{x}_{m,i}^{p}\right]_k \right|^{-2+K-N}}{(N-K)!} \right) \left(\left(\frac{c^2}{\sigma_k^2} \right)^{\frac{1}{2}(K-N-1)} \right) \right.$$
$$\left(\left(c^2 \left| \left[\mathbf{x}_{m,i}^{p}\right]_k \right| \right) \Gamma\left(\frac{1}{2}(N-K+1) \right) {}_1\mathrm{F}_1 \left(\frac{1}{2}(N-K+1), \frac{1}{2}, \frac{K^2\sigma_k^2}{2c^2 \left| \left[\mathbf{x}_{m,i}^{p}\right]_k \right|^2} \right) \right.$$
$$\left. \left. - \sqrt{2} K c \sigma_k^2 \Gamma\left(\frac{1}{2}(N-K+2) \right) {}_1\mathrm{F}_1 \left(\frac{1}{2}(N-K+2), \frac{3}{2}, \frac{K^2\sigma_k^2}{2c^2 \left| \left[\mathbf{x}_{m,i}^{p}\right]_k \right|^2} \right) \right) \right).$$

2. ZF: very accurate approximation of the average rate at the kth user in conventional transmission without RS, using ZF precoding can be written as

$$R_k^{NoRS} = \log_2 M - \frac{1}{M^N}\sum_{m=1}^{M^N} \log_2 \sum_{i=1}^{M^N} e^{-\frac{\left| \sqrt{tP}\beta_p \left[\mathbf{x}_{m,i}^{p}\right]_k \right|^2}{2\sigma_k^2}}$$

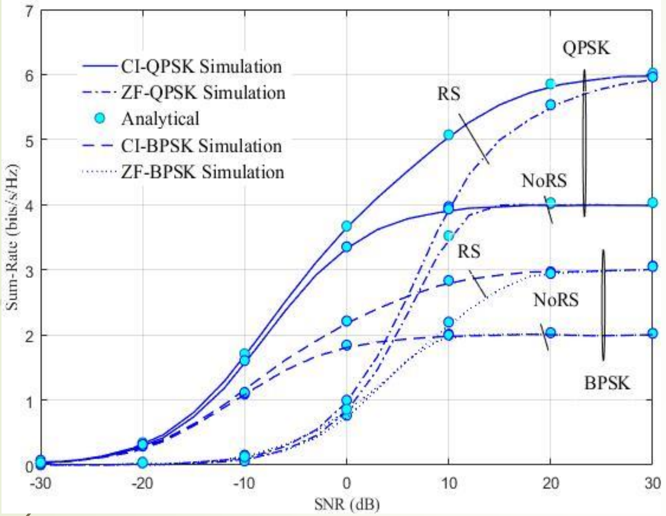
Power Allocation

- The optimal value of t that achieves optimal sum-rate can be derived by maximizing the achievable sum-rate expression.
- However, due to the complicated rate formula in the considered scenarios, the best value of t is obtained by using search algorithm techniques.
- In order to reduce the complexity, the sub-optimal solution studied in Gaussian signalling, can be used also in finite alphabet scenarios. In this solution, we allocate a fraction t of the total power for the private messages to achieve same sum-rate as the conventional techniques with full power. Then, the remaining power is allocated for the common message. The sum-rate payoff of the RS scheme over the NoRS can be determined by

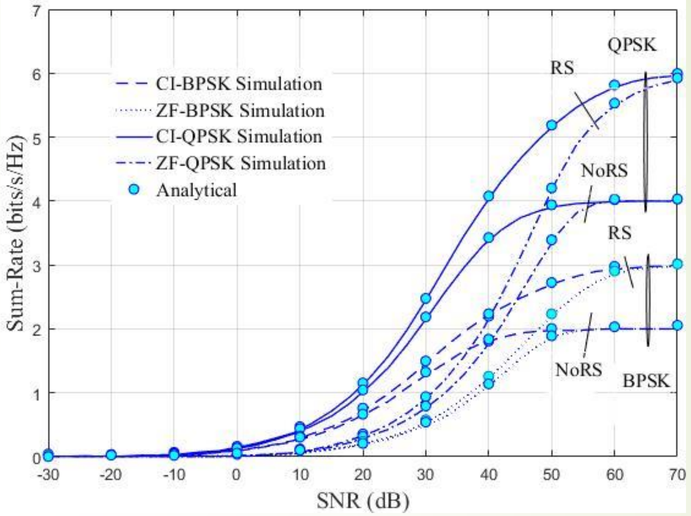
$$\Delta R = R_c + \sum_{k=1}^{K} \left(R_k^p - R_k^{NoRS}\right)$$

Consequently, the power ratio t that achieves the superiority can be obtained by satisfying the equality, $R_k^p = R_k^{NoRS}$.

Numerical Results



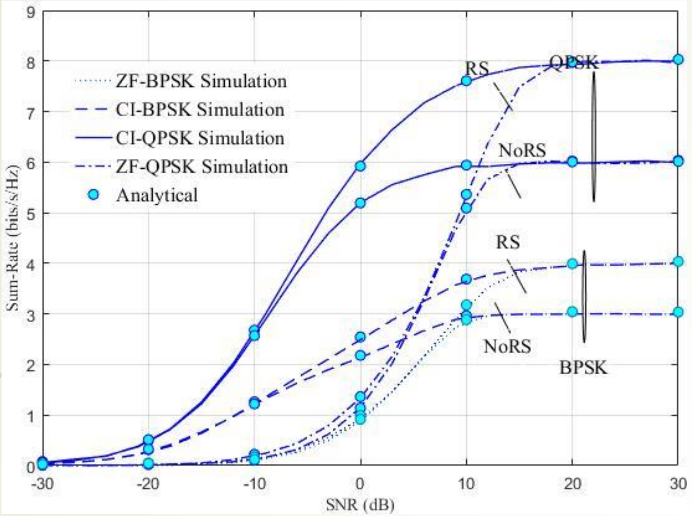
(a) Sum-rate versus SNR, when $d_1 = d_2 = 1$m



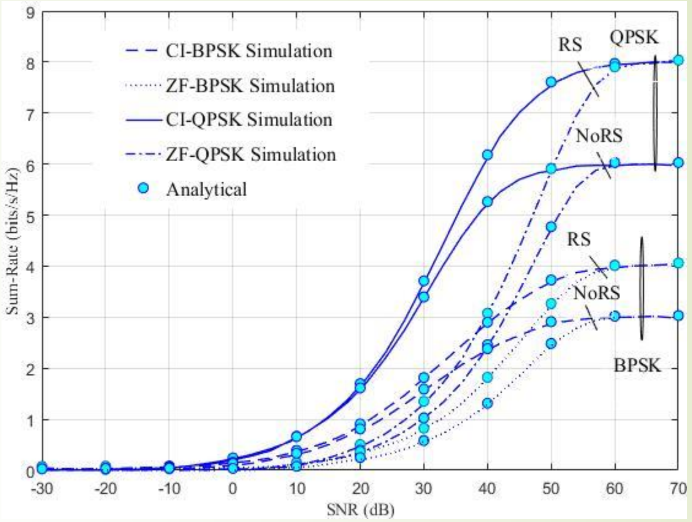
(b) Sum-rate versus SNR, when the users are randomly distributed.

Figure 1: Sum-rate versus SNR for RS and NoRS with different types of input, when N = 3 and K = 2.

- RS scheme enhances the sum-rate of the considered system and CI precoding technique outperforms the ZF technique for a wide SNR range.



(a) Sum-rate versus SNR, when d1 = d2 = d3 = 1m.



(b) Sum-rate versus SNR, when the users are randomly distributed.

Figure 2: Sum-rate versus SNR for RS and NoRS with different types of input, when N = 4 and K = 3.

- It is clear that increasing number of the users K and/or number of the antennas N results in enhancing the achievable sum-rate in the all considered scenarios.

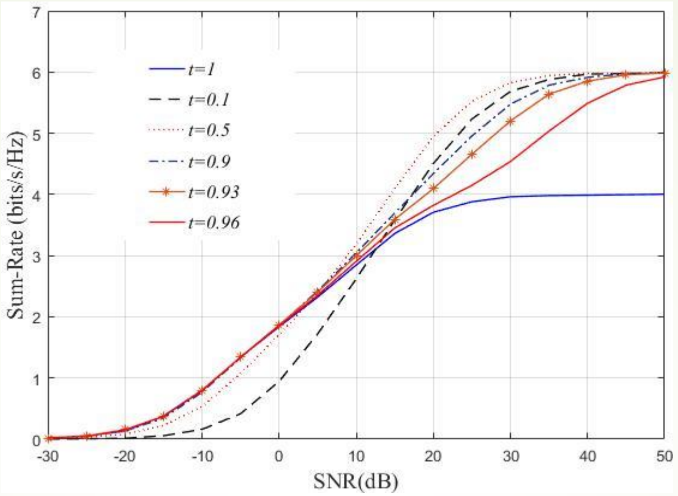


Figure 3: Sum-rate versus SNR for various values of t.

- The optimal value of the power fraction t at low SNR is approximately one, which means that broadcasting the common message is not beneficial, and the RS degenerates to NoRS.
- At high SNR the optimal value of t is less than one, $t < 1$, which indicates that we transmit the common message with remaining power beyond the saturation of the private message transmission.

Conclusions

- In this paper we employed the CI precoding technique to enhance the sum-rate performed by RS scheme in MU-MIMO systems under PSK input alphabet.
- In light of this, new analytical expressions for the average sum-rate have been derived for, CI precoding technique, and ZF precoding technique in RS and NoRS scenarios.
- In addition, power allocation scheme that can achieve superiority of RS over the NoRS in finite alphabet systems was investigated.

- The results demonstrated that, the RS scheme enhances the sum-rate of the considered system and tackles the sum-rate saturation occurred in the communication systems with PSK signalling.

- RS with CI has greater sum-rate than RS with ZF and NoRS transmission techniques.

- In addition, increasing number of BS antennas and/or number of users enhances the achievable sum-rate.

- The optimal value of the power fraction t at low SNR is approximately one and at high SNR the optimal value of t is less than one, which indicates that the common message is transmitted with the remaining power beyond the saturation of the private message transmission.

Thank You

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